6th Canadian Conf. Earthquake Engineering / Toronto / 1991 / 6ème Conf. Canadienne Sismique

# Simplified three-dimensional analysis of concrete gravity dams

A. El-Nady[I], A. Ghobarah[II], and T.S. Aziz[II]

## ABSTRACT

A new procedure is developed for the analysis of concrete gravity dams. This procedure has the capability of including the three dimensional effect with the simplicity of a typical two dimensional analysis. The effect of the adjacent monoliths is represented by interaction shear forces. The results of this new approach were compared to that of a typical three dimensional analysis and high accuracy was obtained. The main advantage of this procedure is the computer cost saving and the simplicity of dealing with beam elements as well as including the interaction of the adjacent monoliths during the ground motion.

## INTRODUCTION

The possiblity of catastrofic failure of concrete gravity dams due to earthquakes presents a hazard for life as well as substantial economic losses in seismically active regions. Existing dams were designed using the equivalent static method which neglects the dynamic properties of the dam and the exciting ground motion. Most of the previous studies on gravity dams used a two dimensional planar model to represent the structure (Chopra 1980 and 1984). Such a two dimensional analysis of the dam is based on the assumption that during an earthquake the expansion joints fail and the monoliths vibrate independently. However, this is not the case in most of the gravity dams where keyed expansion joints between the monoliths are used. A three dimensional analysis of the dam neglecting the expansion joints and simplifying the dam geometry as a rectangular cross- section was performed by Rashed (1985). He indicates that a reduction up to 50% of the response over the two dimensional analysis could be obtained depending on the length/height ratio of the dam. Although many approximations have been introduced in the study, it shows the importance of the third dimension of the dam structure. As the traditional three dimensional analysis of dams, based on the theory of elasticity, is expensive and practically impossible to manage, a new approximate model needs to be developed. This model should save time and effort and at the same time gives reliable results.

Concrete gravity dams in general consist of a number of monoliths (about 15 m wide each) built across the river and separated by expansion joints. These monoliths are fixed at the ground and are subjected to vertical and lateral in plane loads. Depending on the type of the expansion joint used

---

I Research Assistant, Civil Eng. Dept., McMaster University, Canada
II Professor, Civil Eng. Dept., McMaster University, Canada

in the dam, shear forces may develop among the monoliths. however, as the expansion joints are designed to allow for the axial displacement of the monoliths, generally no axial forces are expected. The idea of the new model is to represent the effect of adjacent monoliths, on the one under consideration, by shear forces transmitted through the expansion joints. Therefore, each monolith is considered to be fixed to the ground and connected to other monoliths by links. These links can only transmit shear forces between monoliths and their properties depend on the type and behaviour of the expansion joint used in the dam.

In fact, the relative lateral displacement of the monoliths is the main factor affecting the shear forces developed across the expansion joint. this concept is implemented in the model by using links to connect the monoliths. The forces developed in these links depend on the relative displacement of monoliths.

A plane stress analysis is the most suitable model for each monolith. However, as the number of monoliths in each dam is large, using a finite element model for each monolith will need a large number of degrees of freedom. As a result and as a further simplification, it is suggested that a beam model is used to represent the monoliths for the purpose of calculating the relative displacements among the monoliths. This model provide a great saving in the number of degrees of freedom without loss of accuracy. It provides the capability of including any effects which may change along the longitudinal axis of the dam as long as it can be incorporated in the beam model.

## ANALYSIS PROCEDURE

A concrete gravity dam is normally constructed with joints in order to prevent tensile longitudinal cracks caused by chemical or thermal contraction of concrete and to facilitate the construction work. As each monolith has its own vibration characteristics, they tend to vibrate in different manners. As a result, the motion of the dam may become discontinuous at every monolith causing the water stop plate to break. In order to prevent discontinuous movement between adjacent monoliths, keys are introduced at the joint between the monoliths. Keys also serve the purpose of distributing the stresses in the concrete three dimensionally.

The simplified procedure introduced in this study considers all the monoliths along the dam axis. Each monolith is represented by an equivalent cantilever beam fixed at the ground level and connected with other monoliths by horizontal beams. As the depth / span ratio of the equivalent cantilever beams is high, the effect of shear deformations is included in the procedure. The horizontal beams represent the expansion joints among the monoliths and its properties should be evaluated to give the same behaviour as that of the joints used in the actual dam construction. Although, the procedure is developed for the analysis of concrete gravity dams on rigid foundations and subjected to horizontal ground motion, it could be extended to include the flexibility of the soil and other components of the ground motion.

It is assumed that the expansion joints in gravity dams are capable of transmitting shear forces among the monoliths. Only the horizontal shear developed in the beams connecting the monoliths is considered. The properties of these beams are important and should represent the behaviour of the expansion joints. Although in this study, rigid joints have been assumed, it is possible to incorporate other types of joints and to represent any nonlinear behaviour which may occur. To evaluate the equivalent moment of inertia of these beams the following formula is used:

$$I = \frac{L^3}{12E}\left[\frac{1}{\frac{L_1^3}{3EI_1} + \frac{L_1}{GA_r}}\right] \qquad (1)$$

where:

- $I$ is the equivalent moment of inertia for the connecting beams
- $L$ is the length of the connecting beams
- $E$ is the modulus of elasticity of concrete
- $G$ is the shear modulus of concrete
- $L_1$ is the length of monolith
- $I_1$ is the moment of inertia of the monolith
- $A_r$ is the shear area of the monolith

This formula is developed based on equating the lateral deformations within the monolith to that of a cantilever beam with concentrated load at the edge. The purpose of allowing for such deformations is to simulate the longitudinal deflection behaviour of the dam. In fact, as shear forces develop along the edges of each monolith, it is expected that some longitudinal deformations will take place even within each monolith.

Several methods are available for including the hydrodynamic effects in the dynamic response analysis of concrete gravity dams. The equivalent static method, developed by Westergaad's (1933) is the simplest method available. However, this method underestimates the hydrodynamic forces on the dam as it neglects the water compressibility and dam flexibility. Another method is to model the fluid using a continuum approach or a finite element approach. For the purpose of this study, an approximate technique is used to simulate the hydrodynamic effects considering the horizontal component of an earthquake [Chopra, (1978)]. In this method the effect of the fluid interaction is assumed to be the same as added masses on the upstream face of the dam. The main advantages of this method are the saving in calculation time and effort and the fact that no special programming is needed. The main assumption in this approach is that the response of the dam will be mainly due to the fundamental mode of vibration. This is a realistic assumption as the concrete gravity dams are short vibration period structures.

The dynamic analysis procedure uses a displacement finite element formulation for the dam with added masses to approximate the hydrodynamic effects. The equations of motion of the system are:

$$[M^*]\ddot{\underline{v}} + [C]\dot{\underline{v}} + [K]\underline{v} = [M][R]\ddot{\underline{u}}_g + \underline{R}_m \tag{2}$$

$$[M^*] = [M] + [M_a] \tag{3}$$

Where,

$[M]$, $[C]$ and $[K]$ are the mass, damping and stiffness matrices of the dam

- $\underline{v}$ is the vector of nodal displacements
- $[R]$ is the influence matrix for the ground motion components.
- $\ddot{\underline{u}}_g$ is the vector of ground acceleration
- $[M_a]$ is the added mass matrix
- $\omega_s$ fundamental natural circular frequency of dam without water
- $\underline{R}_m$ is the vector of shear forces transmitted through expansion joints

The added mass representing the hydrodynamic effects are estimated by the formula:

$$m_a(y) = \frac{P(y, \omega_s)}{\psi(y)} \tag{4}$$

$$P(y,\omega_s)=\frac{2w}{gH}\sum_{n-1}^{\infty}\frac{I_n}{\sqrt{\lambda_n^2-\frac{\omega_s^2}{c^2}}} \tag{5}$$

$$I_n=\int_0^H \psi(y)\cos\lambda_n y\,dy \tag{6}$$

$$\lambda_n=[(2n-1)\pi]/2H \tag{7}$$

$\psi(y)$ = The shape of the fundamental mode of vibration of the dam without water.
w = Unit weight of water
c = The velocity of sound in water
H = Water depth
g = Gravity acceleration
x,y represent the longitudinal and the vertical spatial coordinates, respectively

The damping ratio for the equivalent system should be modified according to the following formula:

$$\xi_s=\frac{\omega_s}{\omega}\xi \tag{8}$$

Because the dam - reservoir interaction lowers the fundamental natural vibration frequency of the dam, $\omega_s$, the damping ratio for the equivalent system $\xi_s$ is less than the damping ratio $\xi$ for the dam alone (Chopra, 1978).

## DISCUSSION OF RESULTS

The results of the simplified procedure were compared to those obtained by a typical three dimensional analysis for an idealized dam structure shown in figure (1). The concrete in the dam is assumed to be homogenous, isotropic and linear elastic, with the following properties: unit weight = 24.3 kN/m$^3$, shear modulus = 14.74 x 10$^6$ kPa, which corresponds to a modulus of elasticity = 34.45 x 10$^6$ kPa and Poisson's ratio = 0.17. The earthquake ground motion record used in this study is the S69E component of the (1952) Taft earthquake recorded at the Lincoln School Tunnel. This record is considered to be an intermediate frequency earthquake with an estimated dominant circular frequency content of $\omega$ = 12.5 rad/s (Naumoski 1988). The linear analyses of gravity dam response to seismic ground motion were performed using the finite element code SAP IV. Different cases of the ratio $H_1/H_2$ were considered to study the effect of sudden change in span on the accuracy of the model. Figure (2) shows the simplified three dimensional model of this structure.

The natural frequencies of the structure with different $H_1/H_2$ ratios evaluated by the new model as well as the traditional three dimensional finite element analysis are summarized in table (1). It is noted that the accuracy of the first five modes evaluated by the simplified procedure, S.P., is high. The case $H_1/H_2$ = 0.0 represents a uniform soil profile under the dam, i.e. no shear forces transmitted through the expansion joints as the response of each monolith to the ground motion is the same. However, increasing the ratio $H_1/H_2$ leads to an increase in the shear forces among the monoliths. As a part of testing the new procedure the displacements at different points of the crest are compared. The horizontal component of the Taft ground motion is used in this study. Figures (3),(4) and (5) show the time history of the maximum displacements evaluated by the new procedure and the error relative to the traditional three dimensional analysis for the cases $H_1/H_2 = 0.0$, 0.25 and 0.5 respectively. A reasonable agreement between the two approaches is observed in the aforementioned figures.

Figure (6) shows the maximum displacement profile for the dam crest evaluated by the two procedures for the case $H_1/H_2 = 1.0$. Good accuracy is achieved using the new procedure compared to the three dimensional finite element. Figure (7) shows the same relation for the case $H_1/H_2 = 0.5$. It is noted that the results start diverging somewhat as the ratio $H_1/H_2$ increases. For practical purpose the observed accuracy is considered acceptable as the ratio $H_1/H_2$ normally does not exceed 0.5.

## CONCLUSIONS

A new cost-effective procedure for the analysis of concrete gravity dams including the interaction effects of monoliths, is introduced. The procedure is simple, powerful and is demonstrated to give high accuracy compared to typical three dimensional analysis. As a result of this simplification, other parameters which have not been considered before, such as soil profile under the dam, the change of ground motion along the dam axis and the type of the expansion joint used in the analysis can be studied.

## REFERENCES


Chopra, A., 1978.'Earthquake Resistant Design of Concrete Gravity Dams', Journal of the Structural Division, ASCE, Vol. 104, No. ST6, pp. 953-971.

Chopra, A.K., 1987.'Earthquake Analysis, Design and Safety Evaluation of Concrete Dams', 5th Canadian Conference of Earthquake Engineering, Ottawa, Ontario.

Chopra, A.K., Chakrabati, P. and Gupta, S., 1980. 'Earthquake Response of Concrete Gravity Dams Including Hydrodynamic and Foundation Interaction' Report No. UCB/EERC-80/01, Earthquake Engineering Research Center, University of California, Berkely, California.

El-Nady, A., Ghobarah, A. and Aziz, T., 1990.'Effect of High Frequency Earthquakes on The Response of Concrete Gravity Dams', CSCE annual conference, Hamilton, Ontario, Vol. IV-1, pp. 59-71.

Fenves, G. and Chopra, A.K., 1984.' Earthquake Analysis and Response of Concrete Gravity Dams', Report No. UCB/EERC-84/10, Earthquake Engineering Research Center, University of California, Berkeley, California.

Mlakar, P.F., 1987.'Nonlinear Analysis of Concrete Gravity Dam to Strong Earthquake-Induced Ground Motion', Computers and Structures, Vol. 26, No. 1/2, pp. 165-173.

Naumoski, N., Tso, W.K. and Heidebrech, A., 1988.'A Selection of Represntative Strong Motion Earthquake Records Having Different A/V Ratios', EERG Report 88-01, Earthquake Engineering Research Group, McMaster University.

Rashed, A. and Iwan, W., 1984. 'Earthquake Response of Short length Gravity Dams', Proceeding of Seventh World Conference on Earthquake Engineering, California, pp. 355-362.

Table 1. Natural frequencies of the test structure

| Mode No. | $H_1/H_2 = 0.0$ S.P. | $H_1/H_2 = 0.0$ 3-D | $H_1/H_2 = 0.25$ S.P. | $H_1/H_2 = 0.25$ 3-D | $H_1/H_2 = 0.5$ S.P. | $H_1/H_2 = 0.5$ 3-D |
|---|---|---|---|---|---|---|
| 1 | 120.9 | 121.1 | 105.8 | 102.8 | 95.2 | 92.5 |
| 2 | 169.6 | 171.0 | 156.8 | 161.5 | 150.7 | 157.2 |
| 3 | 268.8 | 264.5 | 256.0 | 255.4 | 244.0 | 242.3 |
| 4 | 367.3 | 304.9 | 297.5 | 298.8 | 249.1 | 280.9 |
| 5 | 369.2 | 374.9 | 315.2 | 319.6 | 269.5 | 295.3 |

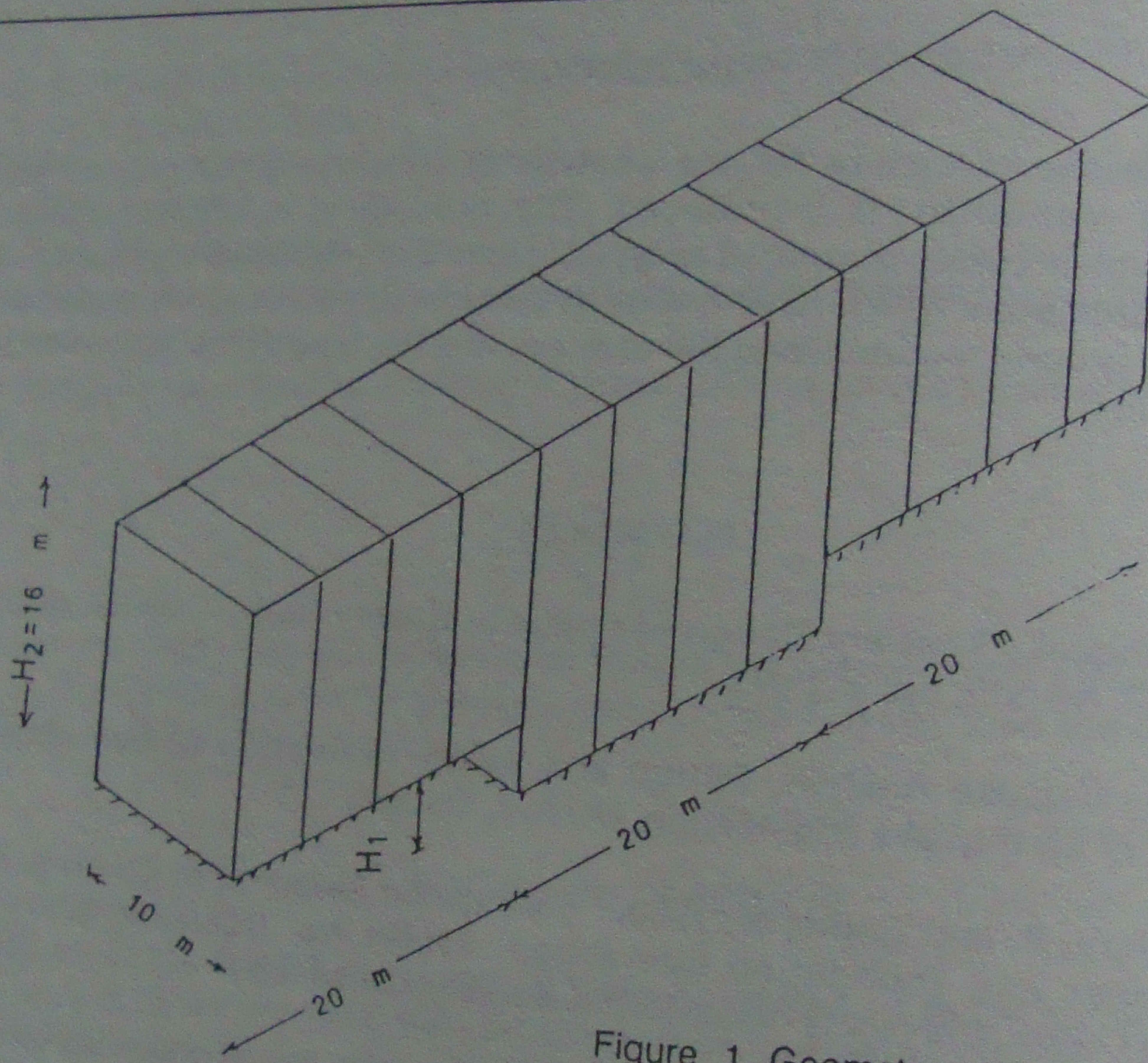



Figure 1. Geometry and dimensions of an idealized dam structure

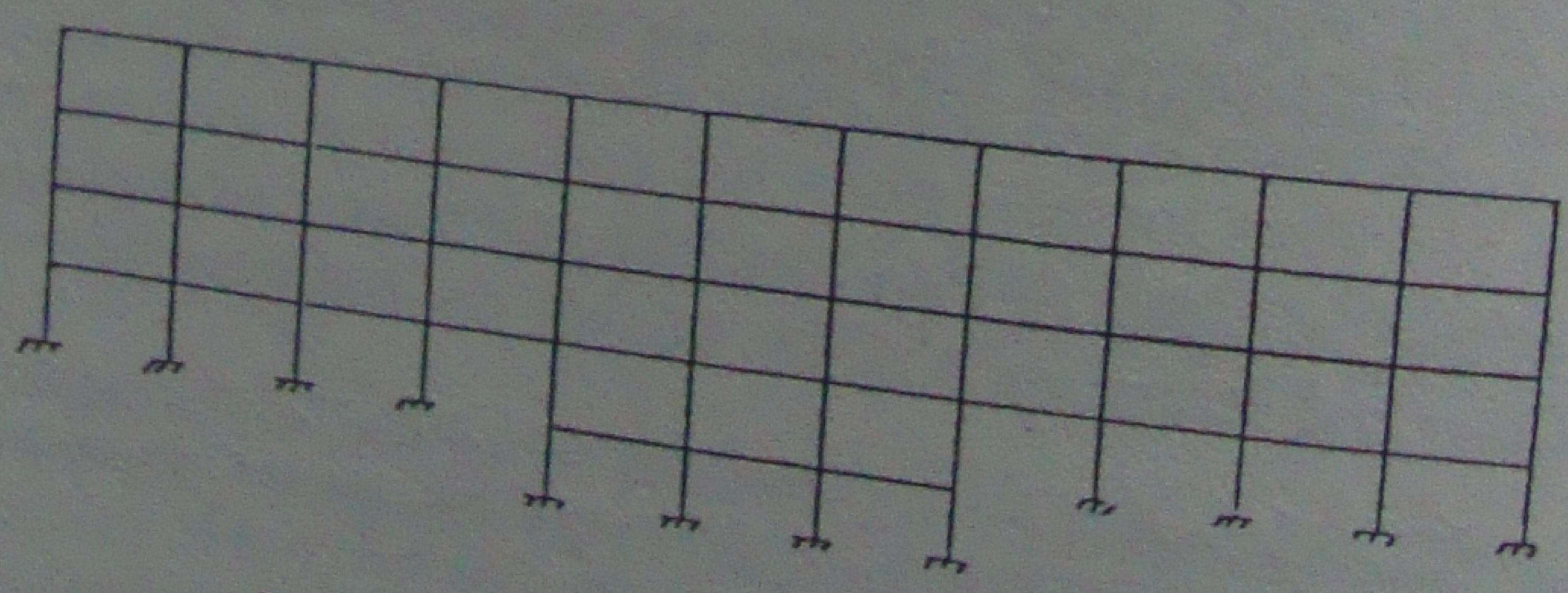
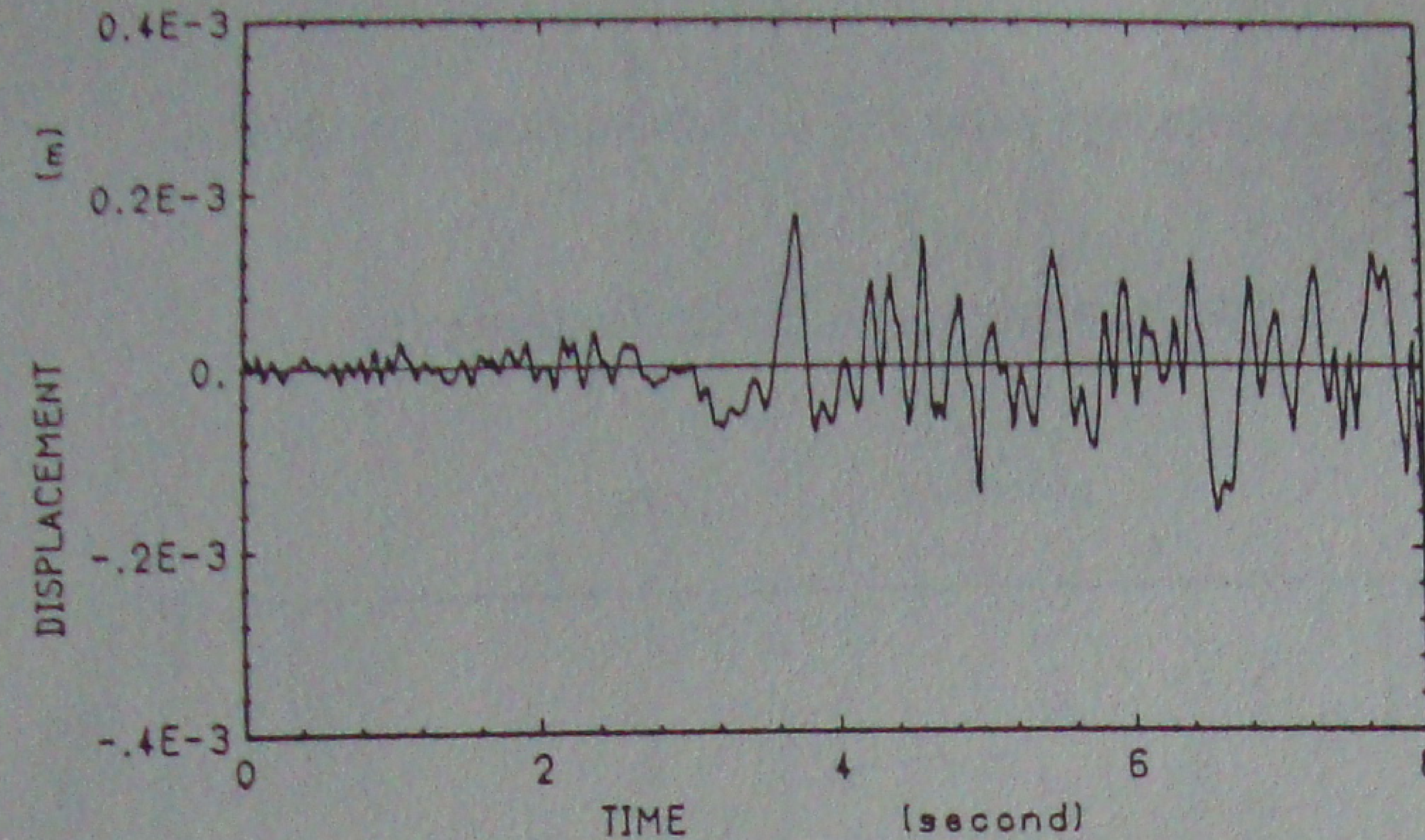
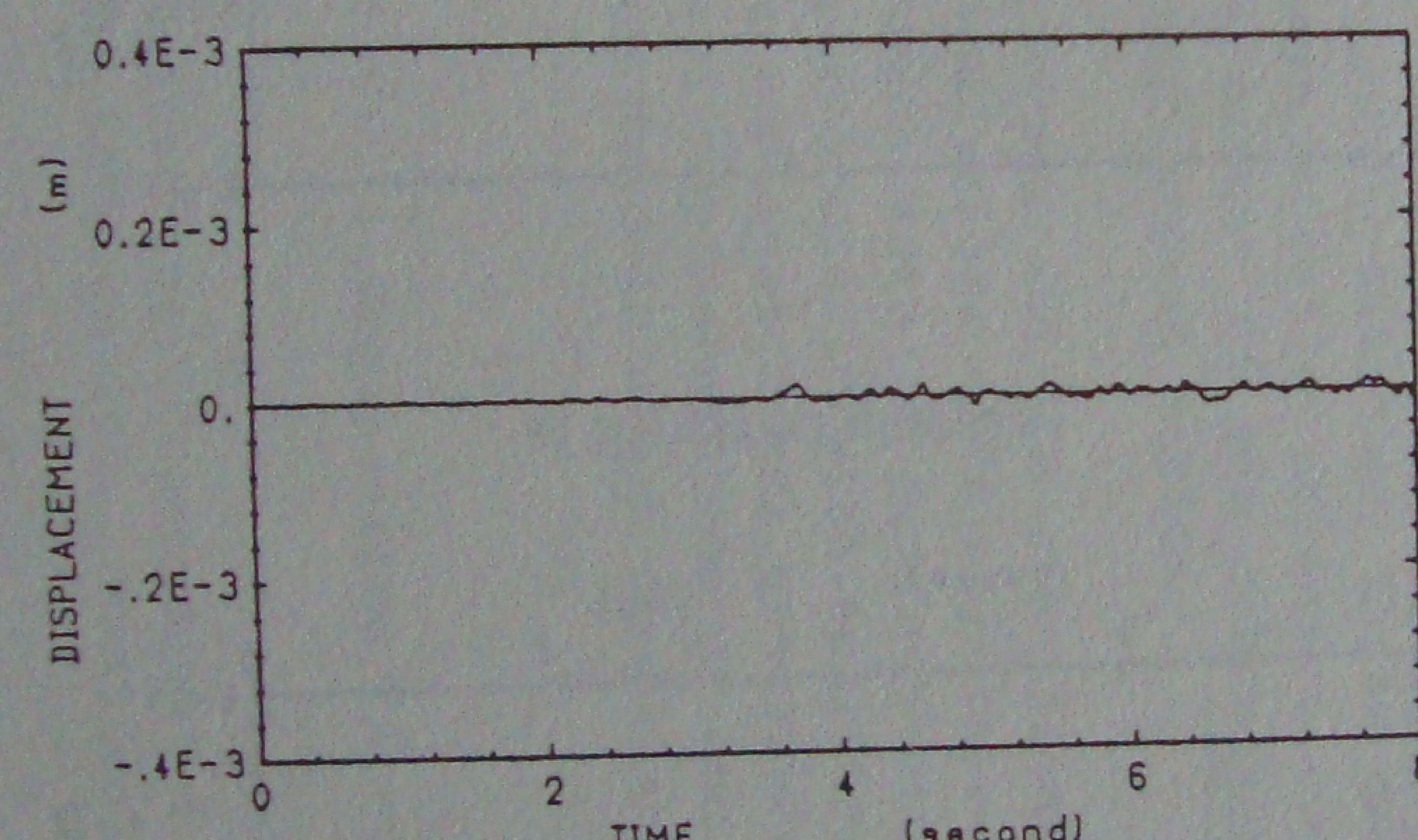


Figure 2. Modelling of the dam structure

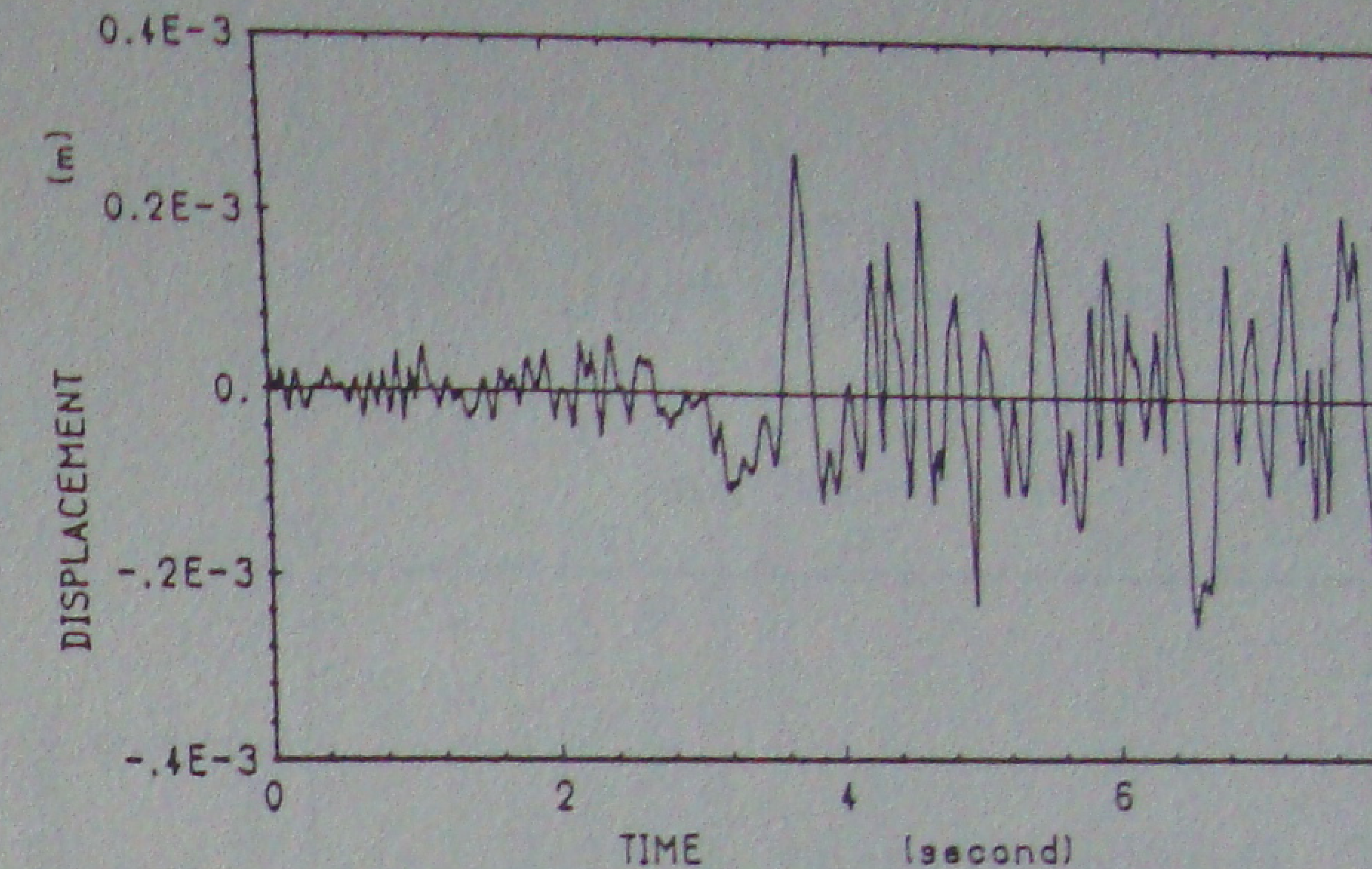



a) Simplified analysis

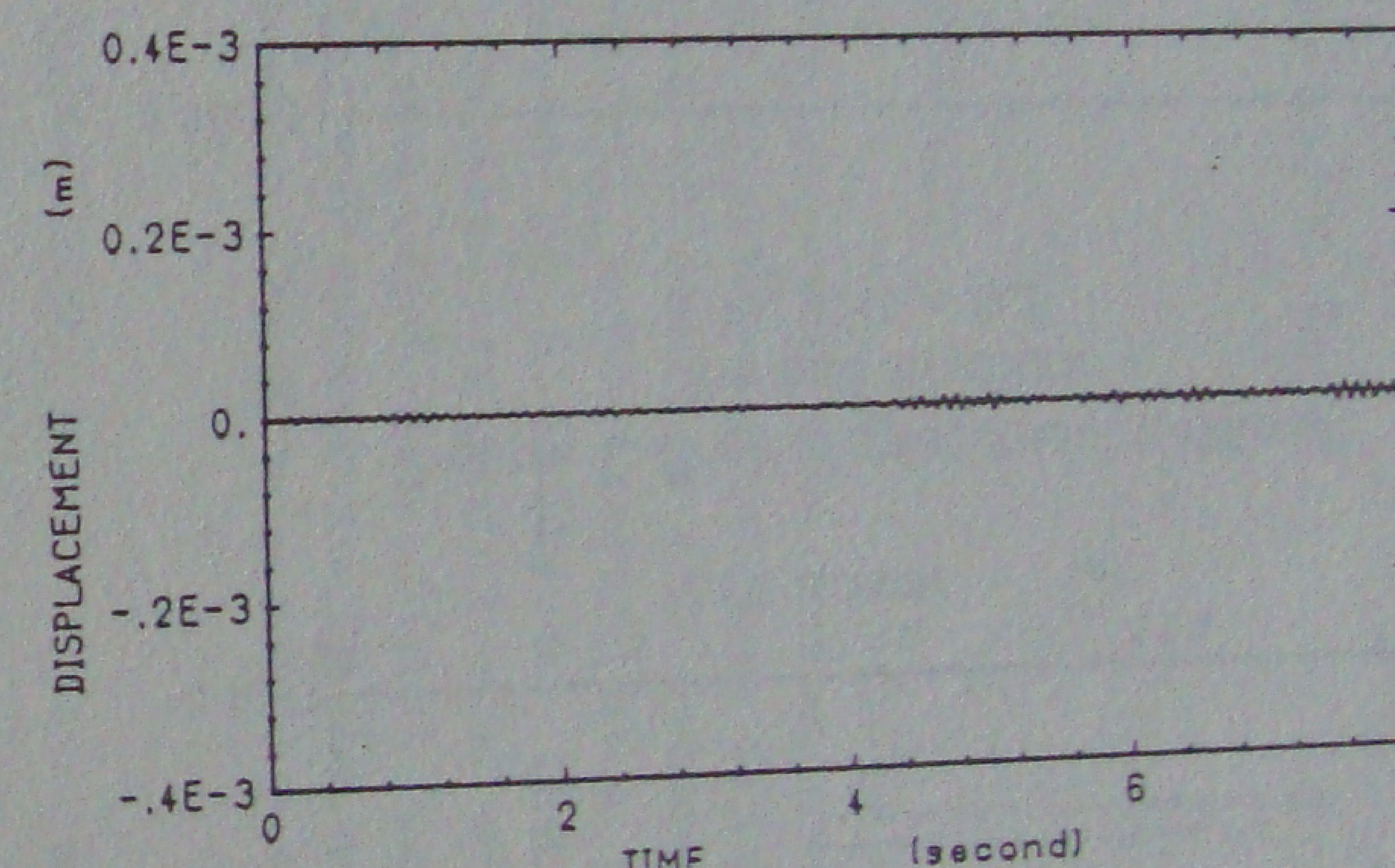



b) Variation from traditional analysis

Figure 3. Time history of the crest displacement
$H_1 / H_2 = 0.0$

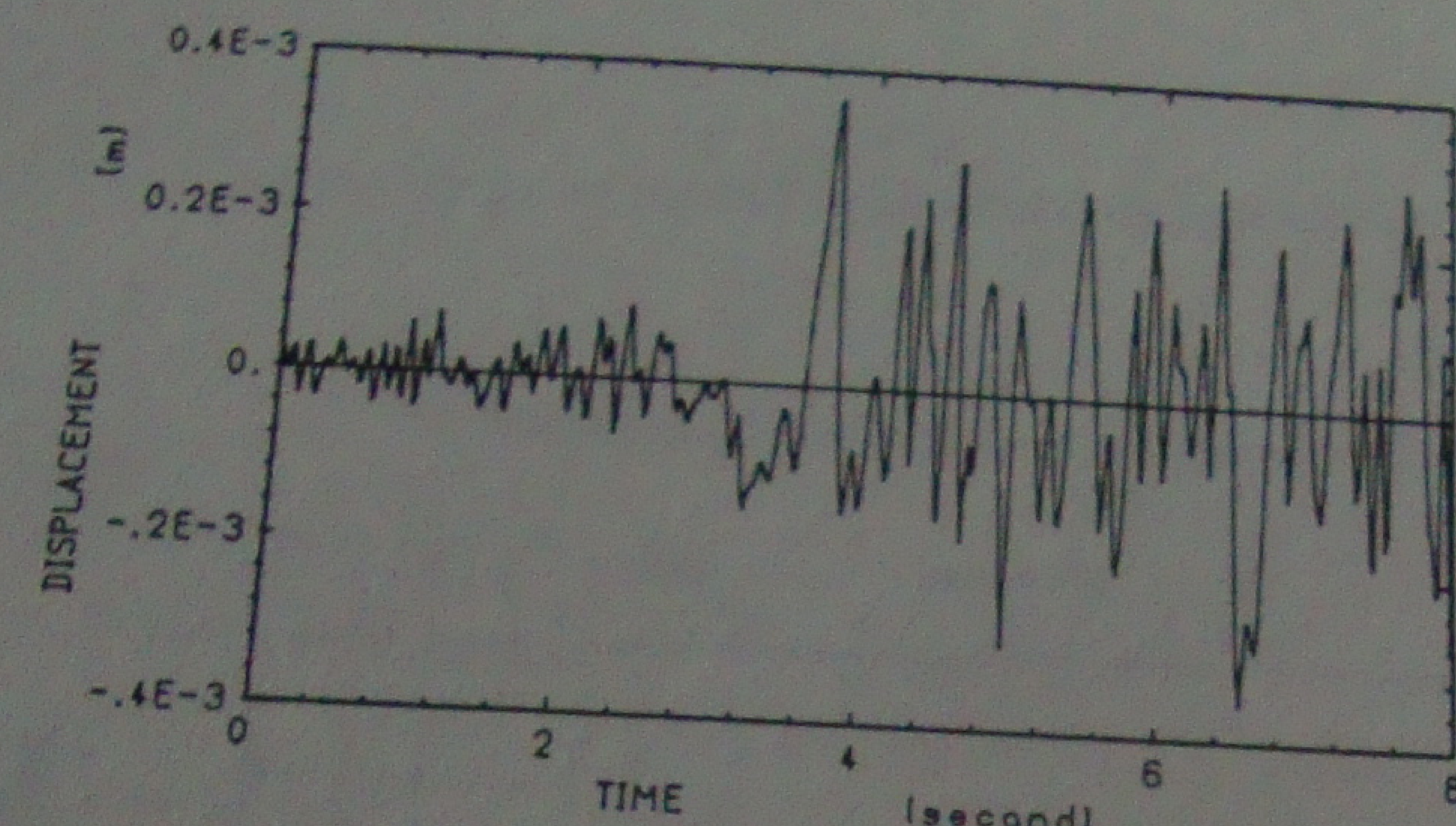



a) Simplified analysis

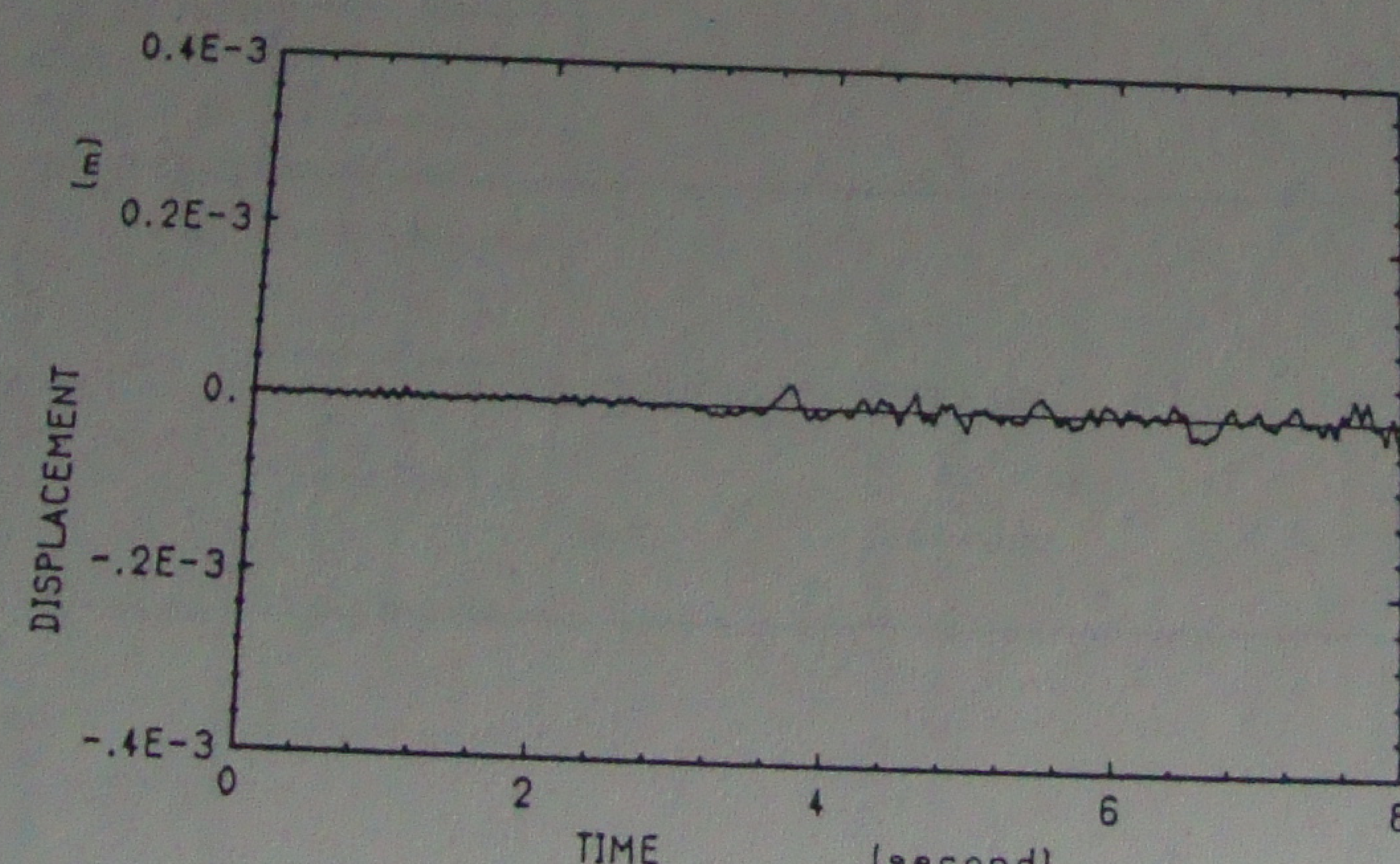



b) Variation from traditional analysis

Figure 4. Time history of the crest displacement
$H_1 / H_2 = 0.25$




a) Simplified analysis




b) Variation from traditional analysis

Figure 5. Time history of the crest displacement $H_1 / H_2 = 0.5$

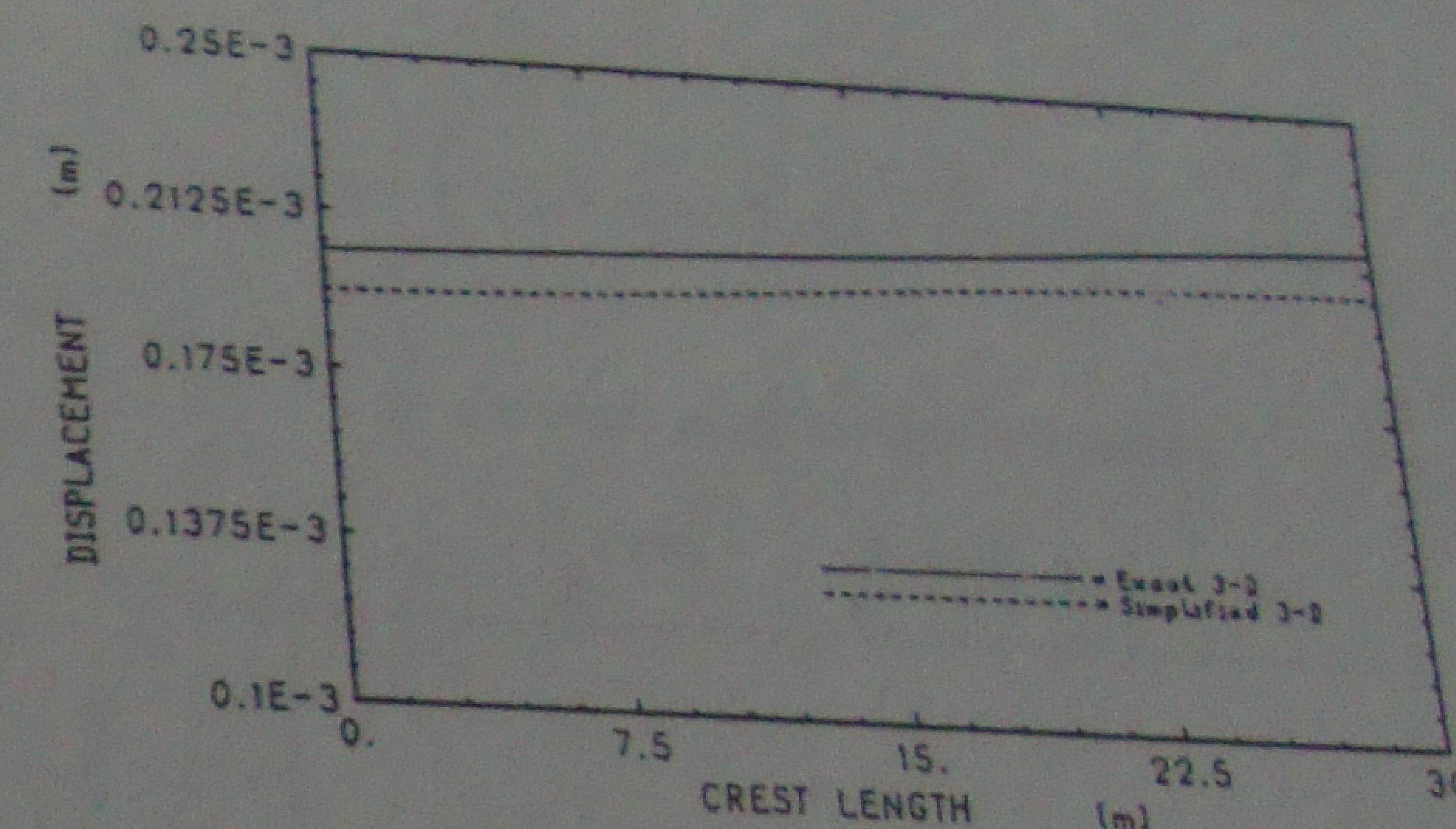



Figure 6. Crest displacement profile of the test structure $H_1 / H_2 = 0.5$

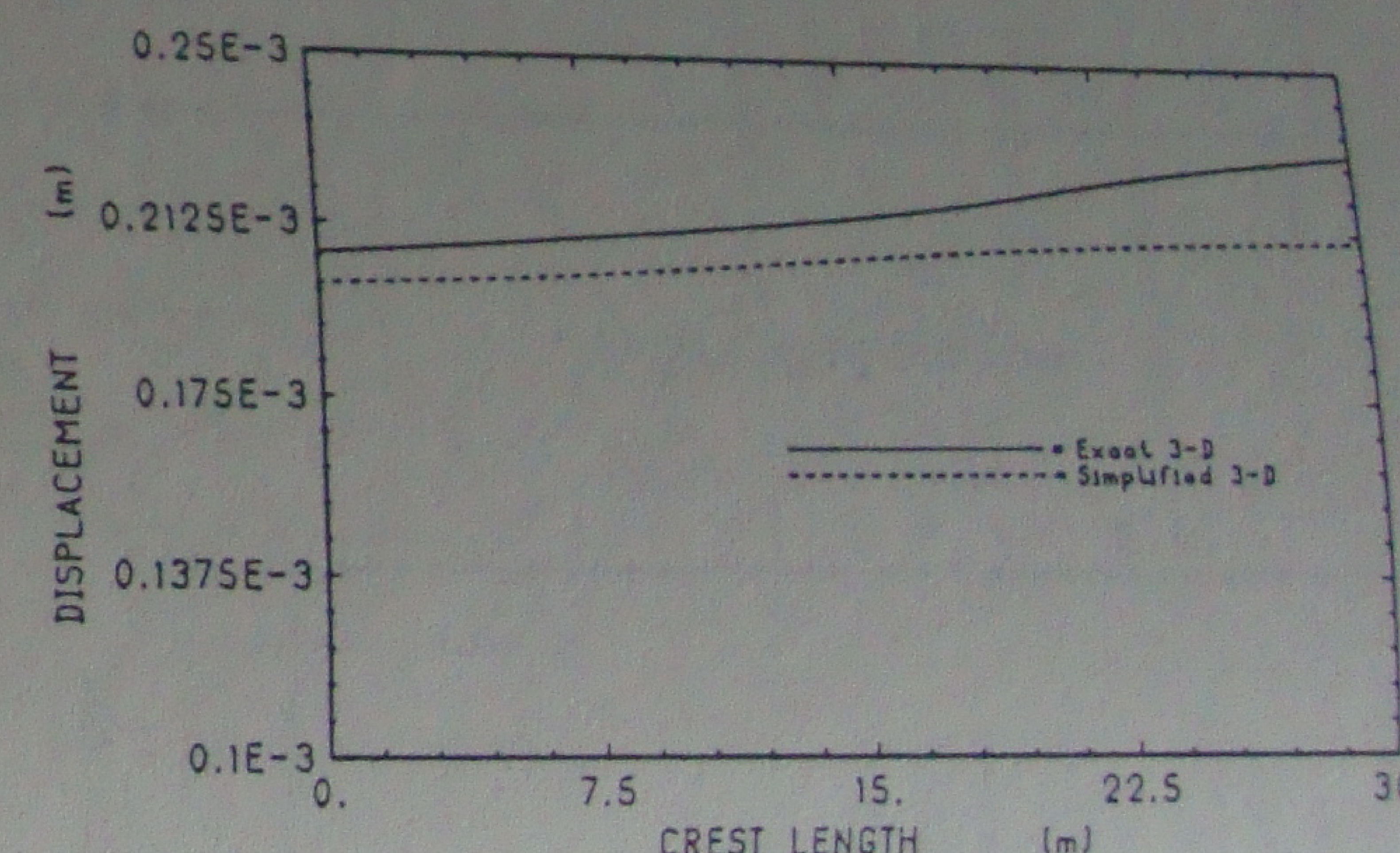



*Figure 7. Crest displacement profile of the test structure $H_1 / H_2 = 1.0$*